[7.6] Homework Solutions

Page: 597

Questions: 3 – 13

Question 3:

Non-permissible value: $t = 0$
Common denominator: $8t$

$$\frac{6}{8} + \frac{6}{t} = 1, t > 0$$

$$8t\left(\frac{6}{8}\right) + 8t\left(\frac{6}{t}\right) = 8t(1)$$

$$6t + 48 = 8t$$
$$48 = 2t$$
$$t = 24$$

It would take 24 min to fill the bathtub using only the hot water tap.

Question 4:

Non-permissible values: $s = -10$ and $s = 0$
Common denominator: $s(s + 10)$

$$\frac{60}{s + 10} = \frac{40}{s}, s > 0$$

$$s(s + 10)\left(\frac{60}{s + 10}\right) = s(s + 10)\left(\frac{40}{s}\right)$$

$$60s = 40s + 400$$
$$20s = 400$$
$$s = 20$$

Aaron's average speed is 20 km/h. Bronwyn's average speed is (20 + 10) km/h, or 30 km/h.

Question 5:

Let t hours represent the time it takes one person to paint the fence using only a brush.
After 2 h, a painter has spray painted $\frac{2}{3}$ of the fence and the person using a brush has painted $\frac{2}{t}$ of the fence.
So, an equation is: $\frac{2}{3} + \frac{2}{t} = 1$
Non-permissible value: $t = 0$
Common denominator: $3t$

$$\frac{2}{3} + \frac{2}{t} = 1, t > 0$$

$$3t\left(\frac{2}{3}\right) + 3t\left(\frac{2}{t}\right) = 3t(1)$$

$$2t + 6 = 3t$$
$$t = 6$$

It would take one person 6 h to paint the fence using only a brush.

Question 6:

Let t hours represent the time it takes Jenny's son to clean out the garage on his own.
After 3 h, Jenny has cleaned out $\frac{3}{5}$ of the garage and Jenny's son has cleaned out $\frac{3}{t}$ of the garage.
So, an equation is: $\frac{3}{5} + \frac{3}{t} = 1, t > 0$
Non-permissible value: $t = 0$
Common denominator: $5t$

$$\frac{3}{5} + \frac{3}{t} = 1$$

$$5t\left(\frac{3}{5}\right) + 5t\left(\frac{3}{t}\right) = 5t(1)$$

$$3t + 15 = 5t$$
$$2t = 15$$
$$t = 7.5$$

It would take Jenny's son 7.5 h to clean out the garage on his own.

Question 7:

Let the volume of bleach added be v litres.
Then, total volume of the solution is $(v + 47)$ litres.

$$\frac{\text{volume of bleach}}{\text{total volume}} = \frac{6}{100}$$

$$\frac{v}{v + 47} = \frac{6}{100}, v > 0$$

$v = -47$ is a non-permissible value.
A common denominator is: $100(v + 47)$

$$100(v + 47)\left(\frac{v}{v + 47}\right) = 100(v + 47)\left(\frac{6}{100}\right)$$

$$100v = 6v + 282$$
$$94v = 282$$
$$v = 3$$

To create a solution that is 6% bleach, 3 L of bleach should be added to 47 L of water.

Question 8:

Let the average speed of the boat in still water be s kilometres per hour.
Average speed downstream: $(s + 3)$ km/h
Distance downstream: 10 km
Time downstream: $\frac{10}{s + 3}$ hours
Average speed upstream: $(s - 3)$ km/h
Distance upstream: 4 km
Time upstream: $\frac{4}{s - 3}$ hours
It takes the same time to travel upstream as it does to travel downstream.
So, an equation is: $\frac{10}{s + 3} = \frac{4}{s - 3}$, $s > 3$
$s = 3$ and $s = -3$ are non-permissible values.
A common denominator is: $(s + 3)(s - 3)$

$$\cancel{(s + 3)}(s - 3)\left(\frac{10}{\cancel{s + 3}}\right) = (s + 3)\cancel{(s - 3)}\left(\frac{4}{\cancel{s - 3}}\right)$$

$$10s - 30 = 4s + 12$$
$$6s = 42$$
$$s = 7$$

The average speed of the boat in still water is 7 km/h.

Question 9:

Let one natural number be x. Then the other natural number is $x + 4$.
The reciprocal of the lesser number is: $\frac{1}{x}$
The reciprocal of the greater number is: $\frac{1}{x + 4}$
Their difference is: $\frac{1}{15}$
So, an equation is: $\frac{1}{x} - \frac{1}{x + 4} = \frac{1}{15}$, $x \in \mathbb{N}$
$x = -4$ and $x = 0$ are non-permissible values.
A common denominator is: $15(x)(x + 4)$

$$15(\cancel{x})(x + 4)\left(\frac{1}{\cancel{x}}\right) - 15(x)\cancel{(x + 4)}\left(\frac{1}{\cancel{x + 4}}\right) = \cancel{15}(x)(x + 4)\left(\frac{1}{\cancel{15}}\right)$$

$$15x + 60 - 15x = x^2 + 4x$$
$$x^2 + 4x - 60 = 0$$
$$(x + 10)(x - 6) = 0$$

$x = -10$ or $x = 6$
Since $x \in \mathbb{N}$, $x = -10$ is not a solution.
So, the natural numbers are 6 and $6 + 4$, or 10.

Question 10:

Let t hours represent the time it takes Marcy to build a deck.
Then, the time it takes Marcy's apprentice is $(t + 9)$ hours.
After 20 h, Marcy has built $\frac{20}{t}$ of the deck and Marcy's apprentice has built $\frac{20}{t + 9}$ of the deck.
So, an equation is: $\frac{20}{t} + \frac{20}{t + 9} = 1, t > 0$
Non-permissible values: $t = 0$ and $t = -9$
Common denominator: $t(t + 9)$

$$\frac{20}{t} + \frac{20}{t + 9} = 1$$

$$\cancel{t}(t + 9)\left(\frac{20}{\cancel{t}}\right) + t\cancel{(t + 9)}\left(\frac{20}{\cancel{t + 9}}\right) = t(t + 9)(1)$$

$$20t + 180 + 20t = t^2 + 9t$$
$$t^2 - 31t - 180 = 0$$
$$(t - 36)(t + 5) = 0$$

$t = 36$ or $t = -5$
Since time cannot be negative, $t = 36$
It would take Marcy 36 h to build the deck and it would take Marcy's apprentice 36 h + 9 h, or 45 h to build the deck.

Question 11:

Let the average speed of the car be s kilometres per hour.
Then the average speed of the airplane is $10s$ kilometres per hour.
Distance: 1000 km
Time for car: $\frac{1000}{s}$ hours
Time for airplane: $\frac{1000}{10s}$ hours
It takes the airplane 18 h less than the car to travel this distance.
So, an equation is: $\frac{1000}{s} - \frac{1000}{10s} = 18, s > 0$
Non-permissible value: $s = 0$
Common denominator: $10s$

$$\frac{1000}{s} - \frac{1000}{10s} = 18$$

$$10\cancel{s}\left(\frac{1000}{\cancel{s}}\right) - \cancel{10s}\left(\frac{1000}{\cancel{10s}}\right) = 10s(18)$$

$$10\,000 - 1000 = 180s$$
$$9000 = 180s$$
$$s = 50$$

The average speed of the car is 50 km/h and the average speed of the airplane is 10(50 km/h), or 500 km/h.

Question 12:

Let Ann's average walking speed be s kilometres per hour.
Then her average cycling speed is $4s$ kilometres per hour.
Distance: 6 km
Time cycling: $\frac{6}{s}$ hours
Time walking: $\frac{6}{4s}$ hours
Total time taken is 90 min, or 1.5 h.
So, an equation is: $\frac{6}{s} + \frac{6}{4s} = 1.5, s > 0$
Non-permissible value: $s = 0$
Common denominator: $4s$

$$\frac{6}{s} + \frac{6}{4s} = 1.5$$

$$4\cancel{s}\left(\frac{6}{\cancel{s}}\right) + \cancel{4s}\left(\frac{6}{\cancel{4s}}\right) = 4s(1.5)$$

$$24 + 6 = 6s$$
$$30 = 6s$$
$$s = 5$$

Ann's average walking speed is 5 km/h and her average cycling speed is 4(5 km/h), or 20 km/h.

Question 13:

Let Brandon's average running speed be s kilometres per hour.
Then Henry's average running speed is $(s + 1)$ kilometres per hour.
Distance: 10 km
Brandon's time: $\frac{10}{s}$ hours
Henry's time: $\frac{10}{s+1}$ hours
It took Brandon 2 min, or $\frac{1}{30}$ h longer to finish the race.
So, an equation is: $\frac{10}{s} - \frac{1}{30} = \frac{10}{s+1}, s > 0$
Non-permissible values: $s = 0$ and $s = -1$
Common denominator: $30s(s + 1)$

$$\frac{10}{s} - \frac{1}{30} = \frac{10}{s+1}$$

$$30\cancel{s}(s+1)\left(\frac{10}{\cancel{s}}\right) - \cancel{30}s(s+1)\left(\frac{1}{\cancel{30}}\right) = 30s\cancel{(s+1)}\left(\frac{10}{\cancel{s+1}}\right)$$

$$300s + 300 - s^2 - s = 300s$$
$$s^2 + s - 300 = 0 \quad \text{Use the quadratic formula.}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute: } a = 1, b = 1, c = -300$$

$$s = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-300)}}{2(1)}$$

$$s = \frac{-1 \pm \sqrt{1201}}{2}$$

$s \doteq 16.8$ or $s \doteq -17.8$
Since speed cannot be negative, $s \doteq 16.8$.
Brandon's average running speed is approximately 16.8 km/h and Henry's average running speed is approximately (16.8 + 1) km/h, or 17.8 km/h.